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# The mode of operation of the uranium reactor

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In an earlier article on this subject the author has dealt with the possible future use of the uranium reactor as a source of energy.[1] However, the calculations were made with the values of nuclear constants then available. Now more exact values have been released by the U. S. A. and using these I have found that the conclusions in my previous paper still remain generally valid. Certain minor adjustments must be made, however. More important is the fact that some further interesting results may be derived, which could not in practice be deduced from the preliminary constants, as the results were too dependent even on slight alterations in these constants.

On all questions of basic principles I refer to the previous article. In this paper I shall confine myself to the large homogenous reactor, but similar modifications may easily be made for the lattice reactor. The same notation will be used as in the previous article. The main difference is that U 235 will not only have a cross section $\sigma_f$ for fission but also a cross section $\sigma_a$ for capture. The relation between the total cross section and the cross section for fission will be denoted $1 + \alpha$.

The following notation is used in the paper:

$N_0$ represents the number of fissionable nuclei per cm$^3$ in a new reactor with a multiplication factor (control rods fully withdrawn) $1 + \varepsilon_0$. $N$ represents the number of fissionable nuclei per cm$^3$ in a reactor which has just ceased functioning because the multiplication factor has fallen to unity. $N_m$ represents the number of nuclei per cm$^3$ of the moderator material.

Each nuclear fission will liberate $a$ neutrons, and of these $\mu_r$ will be absorbed by U 238 in the resonance region, irrespective of the age of the reactor, while $\mu$ will be absorbed in the thermal region when the multiplication factor has just reached unity. The value of $\mu$ in a new reactor with multiplication factor $1 + \varepsilon_0$ is $\mu_0$. $\mu_s$ is the mean value of the thermal absorption by the U 238 from the instant when the reactor is started until it ceases to work because the multiplication factor, control rods withdrawn, has diminished to unity.[2] The harmful absorption in the moderator and other structural parts of the reactor is denoted $\nu$ and $\nu_0$ respectively. $sN_0$ represents the total number of nuclei that disintegrate during the entire life of the reactor. $\sigma_m$ is the nuclear cross section for the absorption in the moderator.[1] $\sigma_c$ is the thermal nuclear cross section for absorption by U 238. $\sigma_a$ is the thermal cross section for capture by U 235 and *Pu*. $\sigma_f$ is the thermal cross section for fission in U 235 and *Pu*.

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[1] Arkiv för fysik, Bd 2 nr 4 (Article submitted in December, 1949).

[2] In the previous paper an unnecessary approximation was made, in putting $\mu_s = \mu$. This has been avoided in the present paper.

According to the released data the constant $a = 2.5$. The fast fission effect, which may be regarded as a kind of delayed neutrons, increases the value by about 3 % to $a = 2.58$. (At the same time one has to reckon with an approximate 3 % increase in the energy released by each U 235 fission.) $\sigma_f = 545$ barns, $\sigma_a = 100$ barns, $\sigma_{fa} = 645$ barns, $\sigma_c = 2.6$ barns.

Referring to the previous article, the following equations may then be written. It is necessary to remember, however, that for one neutron causing fission in U 235, $1 + \alpha = 1.18$ neutrons will be absorbed in U 235, one neutron causing fission and $\alpha = 0.18$ contributing to a new, useless nucleus of U 236. In order to reach the breeding limit it is not sufficient to have one neutron absorbed in U 238 for each fission in U 235. Instead, $1 + \alpha = 1.18$ neutrons must be absorbed if the quantity of fissionable matter is not to be reduced.

$$N_0 - N = s N_0 (1 + \alpha) - (\mu_r + \mu_s) s N_0$$

$$s = \frac{1}{1 + \alpha - \mu_r - \mu_s} \left[ 1 - \frac{N}{N_0} \right] \tag{1}$$

$$a = 1 + \alpha + \mu_r + \mu + \nu = (1 + \varepsilon_0)(1 + \alpha) + \mu_r + \mu_0 + \nu_0 \tag{2}$$

$$\mu = \frac{140\, \sigma_c}{140\, \sigma_c + \sigma_{fa} \dfrac{N}{N_0} + \sigma_m \dfrac{N_m}{N_0}} (a - \mu_r) \tag{3}$$

$$\nu = \frac{\sigma_m \dfrac{N_m}{N_0}}{140\, \sigma_c + \sigma_{fa} \dfrac{N}{N_0} + \sigma_m \dfrac{N_m}{N_0}} (\alpha - \mu_r) \tag{4}$$

$$1 + \alpha = \frac{\sigma_{fa} \dfrac{N}{N_0}}{140\, \sigma_c + \sigma_{fa} \dfrac{N}{N_0} + \sigma_m \dfrac{N_m}{N_0}} (a - \mu_r). \tag{5}$$

If we put

$$\nu_0 = \frac{\sigma_m \dfrac{N_m}{N_0}}{140\, \sigma_c + \sigma_{fa} + \sigma_m \dfrac{N_m}{N_0}} (a - \mu_r) = k (a - \mu_r) \tag{6}$$

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[1] If the reactor is of limited size, one may approximately take into consideration the influence of the neutrons, lost to the environment, by calculating with a somewhat higher $\sigma_m$.

we obtain

$$\sigma_m \frac{N_m}{N_0} = \frac{140\,\sigma_c + \sigma_{fa}}{\frac{1}{k} - 1}. \tag{7}$$

By dividing equations (3) and (5) we obtain

$$\mu = \frac{140\,\sigma_c}{\sigma_{fa}} \frac{N_0}{N} (1 + \alpha) \tag{3 a}$$

and

$$\mu_s = \frac{1}{2} \frac{140\,\sigma_c}{\sigma_{fa}} (1 + \alpha) \left(1 + \frac{N_0}{N}\right). \tag{3 b}$$

From equations (5) and (7) we get

$$\frac{\sigma_{fa} \frac{N}{N_0}}{140\,\sigma_c + \sigma_{fa} \frac{N}{N_0} + \frac{140\,\sigma_c + \sigma_{fa}}{\frac{1}{k} - 1}} (a - \mu_r) = 1 + \alpha. \tag{5 a}$$

With the numerical values given above we obtain

$$\frac{N}{N_0} = \frac{4.29 + \frac{11.9}{\frac{1}{k} - 1}}{6.45\,(1.40 - \mu_r)} \tag{8}$$

$$\mu_s = 0.3325 + 0.3325 \frac{N_0}{N}$$

$$s = \frac{1 - \frac{N}{N_0}}{1.18 - \mu_r - 0.3325 - 0.3325 \frac{N_0}{N}}. \tag{1 a}$$

As in the previous paper we calculate from equations (8) and (1 a) the values of $\frac{N}{N_0}$ and $s$ for three different values of $k$ and $\mu_r$.

I. $k = 0.107$

$\mu_r = 0 \qquad \frac{N}{N_0} = 0.633 \qquad s = 1.15$

$\mu_r = 0.25 \qquad \frac{N}{N_0} = 0.77 \qquad s = 1.35$

$\mu_r = 0.515 \qquad \frac{N}{N_0} = 1 \qquad s = 1.8 \ [\mu_r + \mu = 1.18]$

II. $k = 0.20$

$\mu_r = 0 \qquad \frac{N}{N_0} = 0.805 \qquad s = 0.45$

$\mu_r = 0.15 \qquad \frac{N}{N_0} = 0.90 \qquad s = 0.20$

$\mu_r = 0.28 \qquad \frac{N}{N_0} = 1 \qquad s = 0$

III. $k = 0.27$

$\mu_r = 0 \qquad \frac{N}{N_0} = 0.96 \qquad s = 0.08$

$\mu_r = 0.05 \qquad \frac{N}{N_0} = 1 \qquad s = 0$

As will be seen, the results do not differ much from the previous paper. The same discontinuity is to be found at the breeding limit. Even if the new reactor is infinitely close to that limit, only slightly more energy than that contained in U 235 can be utilised. However, the lower value of $k$, corresponding to the breeding reactor, has the effect that a new reactor which is at the limit of operation cannot be rebuilt for breeding without considerably greater relative reduction in the harmful absorption than was indicated by the earlier data.

An interesting problem is whether a new reactor, which is close to the operating limit, may be rebuilt for breeding if enriched uranium is used, and how high this enrichment must be. It was difficult to decide this when using the earlier, very approximate nuclear constants, as even a slight variation seriously affected the results.

The simplest way of comparing an enriched reactor with a reactor of natural uranium is to consider the number of nuclei per $\mathrm{cm}^3$ of U 235 and U 238 as being the same as before (and consequently $N_0$ remains the same), but to calculate with a cross section $x\sigma_{fa}$ of U 235, where $x$ is the enrichment factor of the U 235. The enrichment has the effect of altering $k$ to $k_x$ if the same moderator is used (see equation (6)). We may then write equation (5 a) for a reactor operating on the breeding limit ($N = N_0$).

$$\frac{x\,\sigma_{fa}}{140\,\sigma_c + x\,\sigma_{fa} + \dfrac{140\,\sigma_c + x\,\sigma_{fa}}{\dfrac{1}{k_x} - 1}}(2.58 - \mu_r) = 1.18 \qquad (9)$$

$$\mu_r = 1.18 - \frac{140\,\sigma_c}{x \cdot \sigma_{fa}} 1.18 \qquad (10)$$

$$\sigma_m \frac{N_m}{N_0} \text{ is unaltered.}$$

Consequently

$$\frac{140\,\sigma_c + \sigma_{fa}}{\dfrac{1}{k} - 1} = \frac{140\,\sigma_c + x\,\sigma_{fa}}{\dfrac{1}{k_x} = 1}. \qquad (11)$$

With the numerical values given for $\sigma_c$ and $\sigma_{fa}$ we may now obtain $x$ from equations (9), (10) and (11) if $k$ is given. In a reactor which is quite close to the working limit we have found $k$ to be 0.27. This value gives $x = 3.1$. However, such a reactor would hardly work with natural uranium, as $\mu_r$ cannot be made as low as 0.05. We also know that graphite reactors have been made to work and have produced a not insignificant quantity of plutonium. Graphite reactors with natural uranium have therefore probably been operating with a $k$-value not higher than 0.20 (see the tables on page 004). For this value of $k$ we get $x = 2.1$.

However, it should be noted that the value of $x$ obtained in this way is slightly too high. A reactor of natural uranium working close to the operating limit, should be designed with a small $\mu_r$ value, as the multiplication factor and $s$ become larger the smaller $\mu_r$ is. In order to ensure breeding it must be redesigned for $\mu_r = 1.18 - \mu$. For an enrichment of $x = 2$ this means a $\mu_r$ value of 0.85. In order to obtain the higher $\mu_r$ value it becomes necessary to increase the uranium at the expense of the moderator mass. But with less moderator mass, i.e. less $\sigma_m \frac{N_m}{N_0}$, the harmful absorption will be less than before. It is therefore probable that the necessary enrichment is not in excess of 2. On the other hand, the loss of fissionable matter by the chemical processes must also be taken into account. However, if only the fission products are to be removed and the plutonium is not separated from the uranium, it seems probable that the loss of fissionable matter may be kept within close limits.

It should be noted that inaccuracies in the nuclear constants such as $\sigma_c$, $\sigma_{fa}$ or $\frac{\sigma_c}{\sigma_{fa}}$ do not greatly affect the result, whereas even a reduction of two to three percent in the accepted values of $\frac{\sigma_f}{\sigma_a}$ and $a$ would have a rather significant influence.[1]

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[1] If we, for instance, figure with $a = 2.425 \cdot 1.03 = 2.50$ instead of $a = 2.50 \cdot 1.03 = 2.58$, we obtain at the breeding limit $s = 2.1$ instead of 1.8 and for the necessary enrichment $x = 2.5$ instead of $x = 2$. If, on the contrary, the capture in *Pu* is less than in U 235, the necessary enrichment $x$ might be considerably less than 2.

As I have already shown, a reactor of natural uranium cannot liberate much more energy than that contained in U 235, even if it is working very close to the breeding limit. The calculations also make it improbable that it is at all possible to build a breeding reactor of natural uranium and using graphite or beryllium as moderator mass. Enrichment by separation of U 235 from U 238 is very costly. It would seem at first sight, therefore, that only the energy contained in U 235 could be extracted at reasonable cost.

However, if the reactor is run until it definitely stops, the fission products being removed continuously or at certain intervals, and the reactor is then rebuilt with the new uranium metal enriched with the plutonium left in the first set of uranium rods, it may be possible to run it a little longer the next time and to have a little more *Pu* left. The third time it will therefore be possible to enrich the fresh supply of uranium a little more. If necessary, this operation may be repeated until the reactor has been enriched with fissionable matter to about 100 % (that means about 1.4 % fissionable matter in the uranium metal). It may then be rebuilt for breeding. After this it can be operated indefinitely (provided that the fission products are removed) by successive addition, at first of the stored quantities of degenerated uranium which will last for decades or longer, and later thorium or natural uranium in quantities which only equal the weight of U 238 consumed in the reactor. In this way practically all the energy available in uranium and thorium on earth may be utilized without any expensive separating processes.

In the above I have dealt only with the homogeneous reactor. It has already been mentioned that similar calculations may also be made for the lattice reactor, along the lines indicated in my previous paper. A reactor of natural uranium with graphite or beryllium as moderator can probably not attain a multiplication factor above unity without lumping the uranium. By lumping the metal it is possible to reduce $\mu_r$ and so to increase the multiplication factor. But to reach the breeding limit, $\mu_r$ must be increased, and with the uranium enriched with fissionable matter to 100 % we have found that $\mu_r$ must be about 0.85. Probably it is possible to build a homogeneous reactor with this resonance absorption, and it might also present certain advantages. But it would not be suitable to use graphite as a moderator in this reactor, as the cooling possibilities of the reactor would be very poor. Very efficient cooling could be ensured, however, by building up the reactor from square section beryllium bars alloyed with uranium. The heat conductivity would be very good and the cooling surfaces could be made so large compared with the uranium mass that perhaps an uncompressed gas, such as helium, might be used as a cooling medium. This would vastly simplify the design of the reactor. The main objection is perhaps the high price of beryllium and the difficult metallurgical process.

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